# 5.5 First Differences

Immediately after jumping from an airplane, a skydiver falls toward Earth with increasing speed. How can you tell if the speed is increasing linearly or non-linearly over time?

In Chapter 2: Relations, you learned how to identify linear and non-linear relations through graphing. In this section, you will learn how to use tables of values to identify the type of relation.

## Investigate

**How can you use a table of values to determine if a relation is linear or non-linear?**

**Tools**

- grid paper

**Method 1: Use Pencil and Paper**

**1.** Consider the relation $y = 3x$.

**a)** Copy and complete the table of values.

**b)** Graph the relation.

**c)** Classify the relation as linear or non-linear.

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 3 |
| 2 | |
| 3 | |
| 4 | |

**2. a)** Describe the pattern in the $x$-values.

**b)** Add a third column to your table to record the change in $y$. Calculate each entry by subtracting consecutive values of $y$. The values in the third column are called **first differences**.

**first differences**

- differences between consecutive $y$-values in tables of values with evenly spaced $x$-values

| $x$ | $y$ | First Differences |
|---|---|---|
| 0 | 0 | |
| 1 | 3 | 3 |
| 2 | | |
| 3 | | |
| 4 | | |

$3 - 0 = 3$

**c)** What do you notice about the values in the third column?

**3.** Repeat steps 1 and 2 for each relation using a table of values with the $x$-values shown.

**a)** $y = -2x + 7$

**b)** $y = x^2$

**c)** $y = 2^x$

| $x$ | $y$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

**4.** Consider your results for the four relations. Make an observation regarding linear relations and first differences.

**5.** Use first differences to determine which of these relations are linear and which are non-linear.

**a)**

| $x$ | $y$ |
|---|---|
| 0 | 7 |
| 1 | 3 |
| 2 | −1 |
| 3 | −5 |
| 4 | −9 |

**b)**

| $x$ | $y$ |
|---|---|
| 0 | −3 |
| 2 | −1 |
| 4 | 3 |
| 6 | 9 |
| 8 | 17 |

**c)**

| $x$ | $y$ |
|---|---|
| −4 | −10 |
| −1 | −5 |
| 2 | 0 |
| 5 | 5 |
| 8 | 10 |

**6. Reflect** Write a rule for using first differences to determine whether a relation is linear or non-linear.

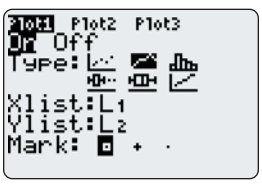
**7. Reflect** Describe how you can tell if the equation of a relation represents a linear relation.

### Method 2: Use Technology

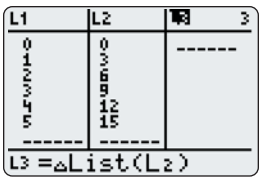
**Tools**
- graphing calculator

**1.** Create a table of values for $y = 3x$ using five values for $x$: 0, 1, 2, 3, 4, 5.
- To clear all lists, press 2nd [MEM] to display the **MEMORY** menu, select **4:ClrAllLists**, and press ENTER.
- To enter the data into the lists, press STAT and select **1:Edit**. Under list **L1**, enter the $x$-values.
- To generate the $y$-values, scroll over to list **L2**, then up to the L2 heading. Type the expression for $y$, substituting the list L1 for $x$. Press 3 × 2nd [L1] ENTER.

**2.** Graph the relation. Is it linear?
- Press 2nd [STATPLOT] to display the **STAT PLOTS** menu. Select **1:Plot1**.
- Press ENTER to select **On**.
- For **Type**:, select line graph.
- Ensure that **Xlist**: is set to **L1** and **Ylist**: is set to **L2**.
- Press ZOOM and select **9:ZoomStat**.



**3. a)** Describe the pattern in the $x$-values displayed in list **L1**.




**b)** Find the differences between successive $y$-values.

- To find the first differences, scroll over and up to the **L3** heading.
- Press [2nd] [LIST] to display the **LIST MATH** menu.
- From the **OPS** menu, select **7:ΔList(**.
- Then, press [2nd] [L2] [)] [ENTER].

**c)** What do you notice about the values in list **L3**?

**4.** Repeat steps 1 to 3 for each relation.

**a)** $y = -2x + 7$

**b)** $y = x^2$

**c)** $y = 2^x$

**5.** Consider your results in steps 2 and 3 for the four relations. Make an observation about linear relations and first differences.

**6.** Use first differences to determine which of these relations are linear and which are non-linear.

**a)**

| $x$ | $y$ |
|---|---|
| 0 | 7 |
| 1 | 3 |
| 2 | −1 |
| 3 | −5 |
| 4 | −9 |

**b)**

| $x$ | $y$ |
|---|---|
| 0 | −3 |
| 2 | −1 |
| 4 | 3 |
| 6 | 9 |
| 8 | 17 |

**c)**

| $x$ | $y$ |
|---|---|
| −4 | −10 |
| −1 | −5 |
| 2 | 0 |
| 5 | 5 |
| 8 | 10 |

**7. Reflect** Write a rule for using first differences to determine whether a relation is linear or non-linear.

**8. Reflect** Describe how you can tell if the equation of a relation represents a linear relation.

## Key Concepts

- To work with first differences, the values of $x$ (independent variable) must change by a constant amount.
- To find first differences, subtract consecutive values of $y$ (dependent variable).

| $x$ | $y$ | First Differences |
|---|---|---|
| 0 | 0 | |
| 1 | 2 | 2 |
| 2 | 4 | 2 |
| 3 | 6 | 2 |
| 4 | 8 | 2 |

$2 - 0 = 2$
$4 - 2 = 2$
$6 - 4 = 2$
$8 - 6 = 2$

- If the first differences of a relation are constant, the relation is linear.
- If the first differences of a relation are not constant, the relation is non-linear.

## Communicate Your Understanding

**C1** For each table of values, decide whether it is possible to use first differences to determine whether the relation is linear or non-linear. Explain your decision.

**a)**

| $x$ | $y$ |
|---|---|
| 0 | 7 |
| 1 | 10 |
| 2 | 13 |
| 3 | 16 |

**b)**

| $x$ | $y$ |
|---|---|
| 0 | 7 |
| 1 | 10 |
| 3 | 16 |
| 6 | 25 |

**C2** Jacob's rate of pay is \$9.50/h. If you made a table of values of Jacob's earnings, how would his hourly wage relate to the first differences?

## Practise

**1.** Look at each equation. Predict whether it represents a linear relation or a non-linear relation. Use a graphing calculator to confirm your answers.

**a)** $y = 5x + 6$
**b)** $y = -3x - 2$
**c)** $y = 4x^2 + 1$
**d)** $y = 10^x$
**e)** $y = -\frac{4}{3}x + \frac{1}{2}$
**f)** $y = \frac{6}{x}$

**2.** Copy each table and include a third column to record first differences. Classify each relation as linear or non-linear.

**a)**

| $x$ | $y$ |
|---|---|
| 0 | 5 |
| 1 | 6 |
| 2 | 8 |
| 3 | 12 |

**b)**

| $x$ | $y$ |
|---|---|
| 3 | −4 |
| 4 | −1 |
| 5 | 2 |
| 6 | 5 |

**c)**

| $x$ | $y$ |
|---|---|
| −1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |

**d)**

| $x$ | $y$ |
|---|---|
| −5 | 8 |
| −3 | 4 |
| −1 | 0 |
| 1 | −4 |

**3.** Each table shows the speed of a skydiver before the parachute opens. Without graphing, determine whether the relation is linear or non-linear.

**a)** There is no air resistance.

| Time (s) | Speed (m/s) |
|---|---|
| 0 | 0 |
| 1 | 9.8 |
| 2 | 19.6 |
| 3 | 29.4 |
| 4 | 39.2 |
| 5 | 49.0 |

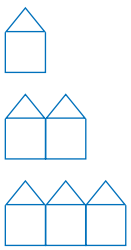
**b)** There is air resistance.

| Time (s) | Speed (m/s) |
|---|---|
| 0 | 0 |
| 1 | 9.6 |
| 2 | 16.6 |
| 3 | 23.1 |
| 4 | 30.8 |
| 5 | 34.2 |

## Connect and Apply

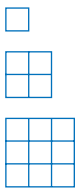
**4.** Use first differences to determine which relations are linear and which are non-linear. Write an equation representing each linear relation. Extrapolate the relation to predict the outcome for the seventh step.

**a)**



| Number of Houses | Number of Segments |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

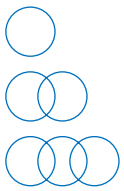
**b)**



| Base Side Length | Total Number of Tiles |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

**5.** Use first differences to determine which relations are linear and which are non-linear. Write an equation representing each linear relation. Extrapolate the relation to predict the outcome for the seventh step.

**a)**



| Number of Circles | Number of Intersection Points |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

**b)**

| Number of Sides | Number of Diagonals |
|---|---|
| 4 | |
| 5 | |
| 6 | |
| 7 | |

**6. Chapter Problem** A pattern is made from toothpicks as shown.

Diagram 1 Diagram 2 Diagram 3

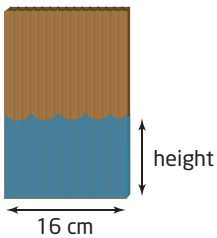
**a)** Create a table comparing the diagram number to the number of toothpicks.

**b)** Use first differences to show that the pattern is a linear relation.

**c)** Write an equation for the relation.

**d)** Extrapolate the relation to predict the outcome for the 10th step.

**7.** A rectangular piece of cardboard is 16 cm wide. It is dipped in water and is wet from the bottom up.



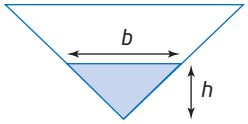

**a)** Create a table comparing the height of the wet cardboard to its area as the height increases from 0 cm to 10 cm.

**b)** Use first differences to determine whether the relation is linear.

**c)** What is the area of wet cardboard if the height is 50 cm?

## Extend

**8.** The triangle's base is twice its height. The triangle is painted from the bottom up.




**a)** Create a table comparing the height of the painted portion to its area as the height increases.

**b)** Use first differences to determine whether the relation is linear.

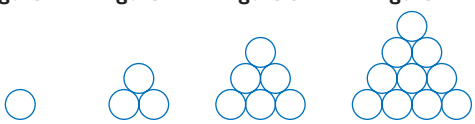
**9.** A class conducted an experiment to see how high a ball would bounce from various heights. The results of one group's experiment are shown in the table.

| Drop Height (cm) | 50 | 100 | 150 | 200 | 250 | 300 |
|---|---|---|---|---|---|---|
| Bounce Height (cm) | 41 | 82 | 125 | 166 | 208 | 254 |

Provide two or more pieces of evidence to show whether this relationship is linear or non-linear.

**10.** The first few figures in a pattern are shown.

Figure 1 Figure 2 Figure 3 Figure 4



**a)** Copy and complete the table.

| Figure Number | Number of Circles in Pattern |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |

**b)** **Use Technology** A graphing calculator can be used to compute first differences. Follow the steps below.

- Enter the values from column 1 (Figure Number) in list **L1**.
- Enter the values from column 2 (Number of Circles in Pattern) in list **L2**.
- Place the cursor on **L3** using the cursor keys.
- Press 2nd [LIST]. From the **OPS** menu, select **7:ΔList(**. Press 2nd [L2] ) ENTER.

What information is in **L3**? Use this information to create a non-linear equation for this pattern.